Aim: Calculate the Exchange Energy, i.e. the energy of a two electron system if the spins are parallel or antiparallel (a priori classically the only difference is due to dipolar coupling).

$\mathcal{H} = J * \hat{\mathbf{S}}^a \cdot \hat{\mathbf{S}}^b$

Calculate $\hat{\mathbf{S}}^a \cdot \hat{\mathbf{S}}^b$ (Quantum mechanical interlude chapter 2.2.1)

Calculate J (chapter 2.3)

Path:

2.2.1.1-2.2.1.3 Quantum Mechanical Interlude:

-For Fermions the total wave function is antisymmetric
(spatial wave function * spin wave function)

-Spin wave function is constructed using Spinor representation

-Calculate $\hat{\mathbf{S}}^a \cdot \hat{\mathbf{S}}^b$ for electrons with spin 1/2

-Two spin wave functions are found:
Singlet state (antiparallel) and
triplet state (parallel and three times degenerate)

-Combine spin wave function with spatial wave function

Path:

2.3 Calculate Exchange interaction:

-Parametrize the difference in energy between the parallel and antiparallel alignment

-Obtain relation for J = ... depending on the spatial wave functions of both electrons.

Main result:

We can calculate the difference between the state for parallel and antiparallel alignment:

$\mathcal{H} = J * \hat{\mathbf{S}}^a \cdot \hat{\mathbf{S}}^b$ with J=...

This is then used in the micromagnetic model as the Heisenberg Hamiltonian.