Experimentalphysik 5c, Physik der kondensierten Materie WS16/17
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Exercise Sheet 12

Exercise 1: Landé-Factor

Determine the Landé-Factor g_J of an atom using the operator of the magnetic moment

$$\hat{m} = \mu_B\left(g_L\hat{L} + g_S\hat{S}\right) = g_J\mu_B\hat{J}$$

g_L and g_S are the g-factor of the orbital and spin angular momentum. Write the solution as

$$g_J = g_L f_1(L;S;J) + g_S f_2(L;S;J)$$

where f_1 and f_2 are equations, that (only) depend on the quantum numbers L, J, S. Show that this form is equivalent to the form that is given in the lecture. (2P)

Exercise 2: Coupling betweeng magnetic dipoles

Calculate the energy due to the dipole-dipole interaction assuming a distance of 0,3nm and a magnetic moment of $1\mu_B$. How does the energy compare with the thermal energy at room temperature? Would this coupling be useful for a memory device? Can dipolar coupling be responsible for magnetism (compare to typical order-disorder transition (Curie-temperatures) of magnetic materials). (2P)

Exercise 3: Hund's rules and extinction of orbital angular momentum

a. Determine the ground term for the rare earth element ions Cerium Ce^{3+} and Holmium Ho^{3+} using Hund's rules. Calculate the effective number of magnetons $p = g(J(J+1))^{1/2}$. Compare the results to those obtained experimentally. (1P)

b. Calculate the ground term and the effective number of magnetons $p_J = g(J(J+1))^{1/2}$ and $p_S = g(S(S+1))^{1/2}$ for the ions of the iron group Titanium $Ti^{3+}$and Nickel Ni^{2+}. Compare the results to those obtained experimentally. How about the total orbital angular momentum L? (2P)

As seen in b. there is a good agreement of the theoretical and experimental results, if you neglect the orbital angular momentum ($L = 0$). The reason lies in the crystal field which is caused by the ions, and which can strongly influence the 3d-shell energetically (in contrast to the rare earth ions). This leads to an annihilation of the coupling between L and S and the degeneracy of the 2L+1 sublevels.

In an orthorhombic crystal charges of neighbor atoms create an electrostatic potential described by

$$e\varphi = Ax^2 + Bx^2 - (A+B)z^2,$$

where A and B are constants.

c. Show that this polynomial is a solution of the Laplace equation $\nabla^2\varphi = 0$.

The three following p-wave functions describe the ground state of the ion:

$$U_x = xf(r), U_y = yf(r), U_z = zf(r)$$

(1P)

d. Show that these wave functions are orthogonal with respect to the perturbation by the crystal field potential, meaning:

$$\langle U_x|e\varphi|U_y\rangle = \langle U_y|e\varphi|U_z\rangle = \langle U_z|e\varphi|U_x\rangle = 0$$

Proof that the expectation value of the z-component of the angular momentum $L_z = -i\hbar y\frac{d}{dz} + i\hbar z\frac{d}{dy}$ is equal to zero:

$$\langle U_x|L_z|U_x\rangle = \langle U_y|L_z|U_y\rangle = \langle U_z|L_z|U_z\rangle = 0$$

This effect is known as "extinction of the orbital angular momentum". (2P)