

## 3.0 Micromagnetics - a model to describe magnetization

### 3.0.1 Examples of inhomogeneous magnetization

- So far, primarily microscopic models for the exchange interaction have been dealt with.  
→ All spins are preferentially aligned parallel.
- But we have found experimentally that there are inhomogeneous magnetization configurations, why?
- In addition to exchange energy, other energy terms are important, such as the anisotropies that already appeared in the course!

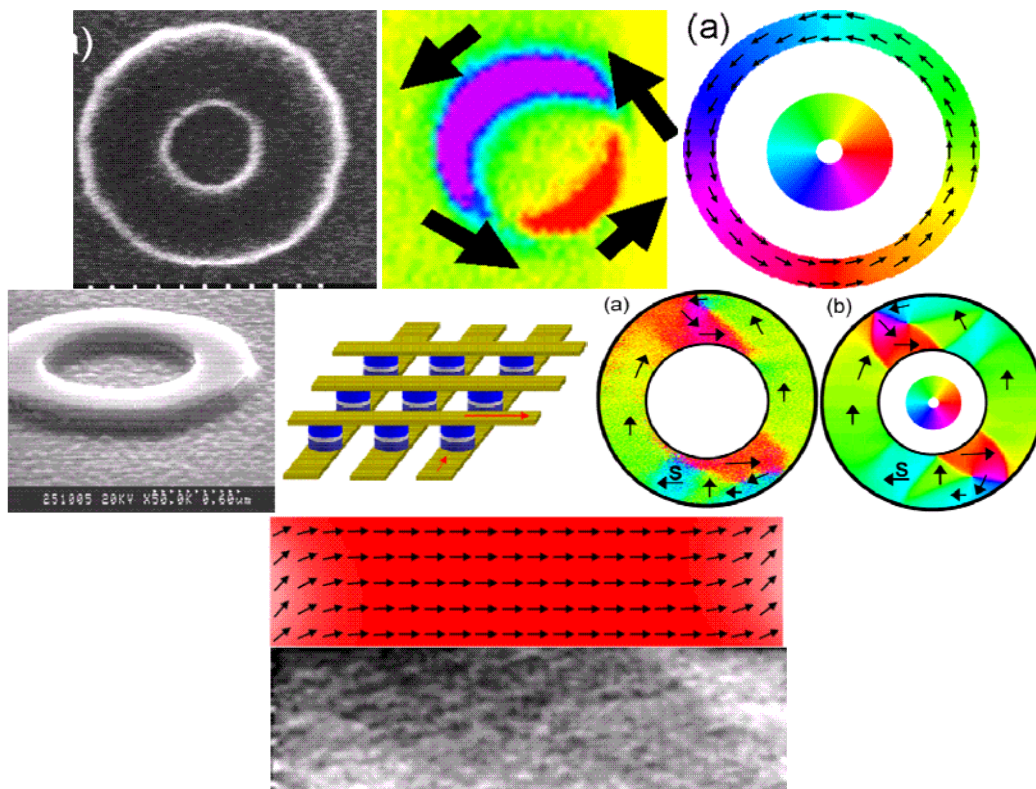


Figure: Inhomogeneous magnetization configurations

- Not only do inhomogeneous magnetization configurations exist, but different configurations can exist even in a given geometry (for instance discs).  
→ History of the system is important.

**Goal:** model for the description of inhomogeneous magnetization and changes of the magnetization.

- Apart from an understanding of the physics, such inhomogeneous configurations are also important for applications (hard drives, etc.)!

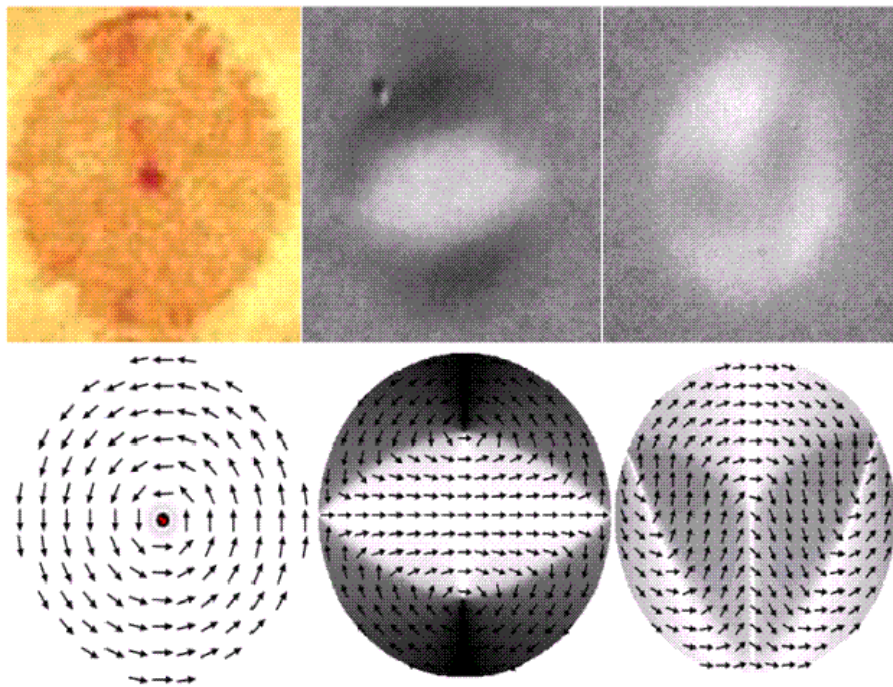


Figure: Different magnetization configurations in the same geometry (disc)

### 3.0.2 Thermodynamics background

- 2<sup>nd</sup> law of thermodynamics: in equilibrium the entropy  $S = \ln W$  will be maximized.
  - This holds in **closed** systems.
  - Experimentally one usually deals with **open** systems.
- Equilibrium is described by the extremum of the appropriate thermodynamic potential!

**Example:** system in temperature bath →  $T = \text{const.}$

- Total entropy of the system + temperature bath is maximum when the free energy of the system  $F = U - TS$  is minimum.

- A magnetic system in a temperature bath:
- Which potential describes the equilibrium?
- 1<sup>st</sup> law of thermodynamics:
- Take  $V = \text{constant}$ ,  $N = \text{constant}$ :

→ Gibbs free energy is the appropriate potential:

**In equilibrium  $G$  has a minimum! This means that for equilibrium states one needs to determine the magnetization configuration that minimizes  $G$ !**

### 3.0.3 Energy terms in the micromagnetic approximation

- Let  $T = 0$  (we neglect temperature effects, which will be dealt with in section 5.6.12).
  - Gibbs free energy:  $G = U - V\mathbf{B}_0\mathbf{M}$  should be minimized.
  - If there is an inhomogeneous magnetization configuration  $\mathcal{M}$ :  $\mathcal{M} = \mathbf{M}(\mathbf{r})$ , then  $G$  has to depend on the magnetization configuration  $\mathcal{M}$ :  $G = G(\mathcal{M})$ .
  - **What does  $G$  look like exactly** (sometimes called Landau free energy  $G_L$ ), so one can calculate it?
  - So far we have used discrete magnetic moments and so one obtains the total energy as a sum over the energy of all the spins.
- Difficult to calculate, because for analytical solutions one would prefer to use continuous systems (calculus of continuous functions is easier than for discrete ones).

- **Micromagnetic approximation** (is used, for instance, to describe low-dimensional systems such as thin films):
- $\mathcal{M} = \mathbf{M}(\mathbf{r})$  is a **continuous function of the position** (vector field).

**Example:**



Figure: Magnetization configuration

$M_x(\mathbf{r}), M_y(\mathbf{r})$  ?

- Micromagnetic approximation is a valid approximation if:
  - i.) System size  $\gg$  atomic distance (no effects due to discrete nature of atoms and electrons).
  - ii.) Quantum effects do not play a role,  $\mathcal{M}$  is classical vector.
  - iii.)  $T \ll T_c$  since  $|\mathbf{M}| = M_s = \text{constant}$

## Summary Energy Terms:

$$G = U(\mathcal{M}) = E_{\text{Zeeman}} \\ = E_{\text{Exchange}} + E_{\text{Aniso}} + E_d + E_{\text{Magnetostriiction}} + E_{\text{Stress}} + E_{\text{Zeeman}} \dots$$

- Thermodynamics: Gibbs free energy  $G$  as the sum of Zeeman, Exchange, Anisotropy, Dipolar, Magnetostriiction, Stress is minimized in equilibrium.

## Further Outlook:

1. Calculate Energy Terms for a given magnetization configuration
2. Find static magnetization configuration that has minimum energy
3. Calculate dynamics of changes of the magnetization configuration when fields are applied or currents injects

All this is generally possible for small elements, which are usually found at surfaces, where special energy terms play a role (surface anisotropy, etc.).

→ First learn something about surface science!

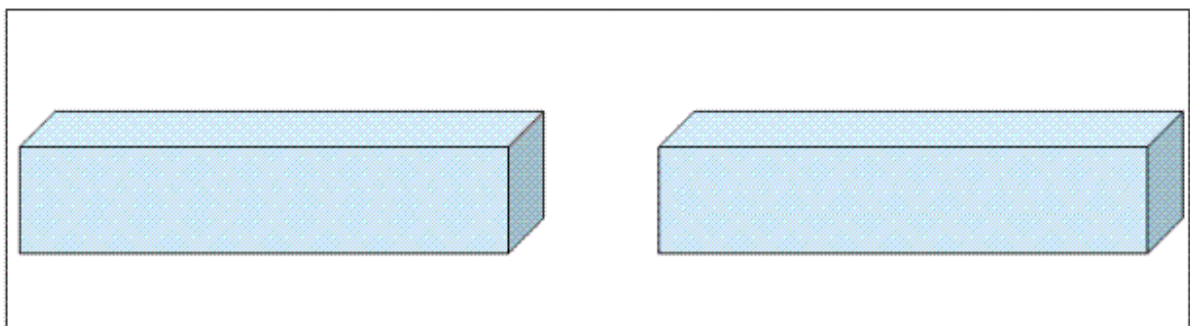
→ Use the knowledge to calculate the energy terms!

### 3.1 Zeeman energy

$$G = U - V\mathbf{B}_0\mathbf{M}$$

(For inhomogeneous magnetization  $\mathbf{M}$ ;  $\mathbf{B}_0$ : Zeeman term)

- Description: The Zeeman term describes the energy of magnetic dipoles in an external homogeneous field (see section 2.1).
- Micromagnetic approximation: For an inhomogeneous magnetization vector field  $\mathcal{M} = \mathbf{M}(\mathbf{r})$  we find the integral over the volume (transition from the sum of discrete spins to the integral of the continuous magnetization):



- Properties: In a homogeneous external field  $\mathbf{B}_0$  the Zeeman energy only depends on the average magnetization!



### 3.2 Exchange energy

$$G = U - V\mathbf{B}_0\mathbf{M} = U(\mathcal{M}) + E_{Zeeman}$$

- In the intrinsic energy  $U$  more energy terms are hidden, such as the exchange energy, which also depends on the magnetization configuration  $\mathcal{M}$ :

$$U(\mathcal{M}) = \text{Exchange}(\mathcal{M}) + \dots$$

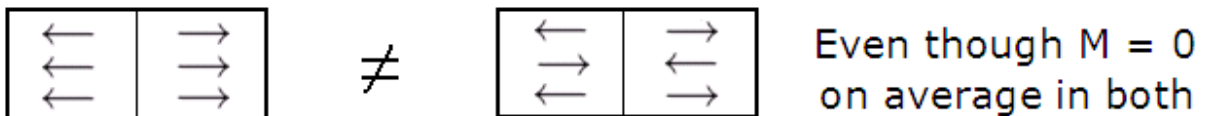


Figure: Different exchange energy for the same average magnetization

- Description: The exchange term describes the exchange interaction between neighbouring magnetic moments.

- Derivation: Heisenberg model for discrete spins (see chapter 4):

Quantum mechanical:

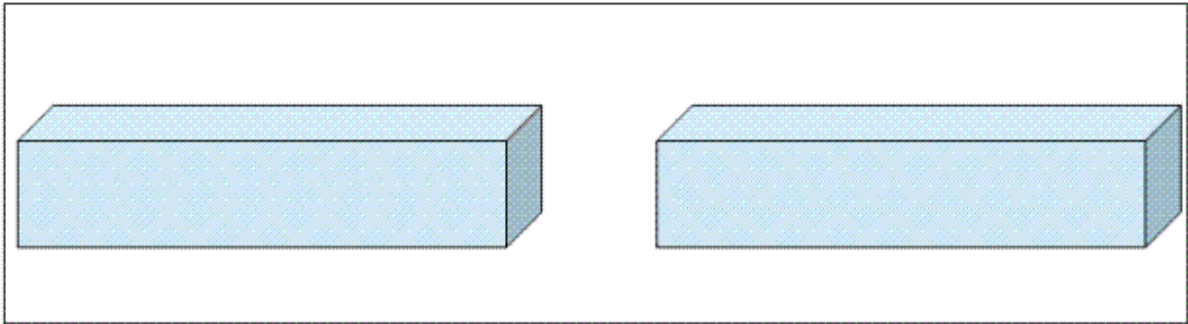
Classical:

$J_{ij} > 0 \rightarrow$  ferromagnet,

$J_{ij} < 0 \rightarrow$  antiferromagnet, (\*)

$J_{ij} \neq 0$  for neighbours and possibly next neighbours

- Micromagnetic Approximation: Transition from a sum of discrete spins to the integral of the continuous magnetization:
- Assumption: Change of direction from one spin to the next is small.  
→ Continuous magnetization can be expanded into a Taylor series.
- With  $A = cJ/2a$ ,  $c$  depends on the crystal structure (number of neighbours, etc. For a cubic crystal (sc crystal structure) we have  $c = 1$ , for bcc:  $c = 2$ , for fcc:  $c = 4$ ).



- Properties:

- Very short ranged (mostly only next neighbour).
- Increases as soon as there is inhomogeneous magnetization.
- Mostly isotropic.
- Strength:  $>10$  Tesla (field that is necessary to align the spin antiparallel).

### 3.3 Magnetic anisotropy energy

$$U(\mathcal{M}) = E_{Exchange}(\mathcal{M}) + E_{Aniso}(\mathcal{M})...$$

- Description: Spin-orbit-coupling → preferred orientations/directions in some systems.

The dependence of the anisotropy energy on the direction of the magnetization can be described phenomenologically.

- Derivation: Spin – orbit coupling favouring certain magnetocrystalline directions.
- Micromagnetic Approximation:  
Transition from the sum of the spins to the integral over **M**:

In 2 dimensions ( $\Theta$  = angle of the magnetization direction in the plane):

### 3.4 Stray field energy

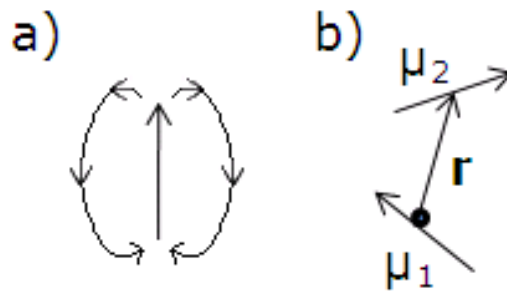
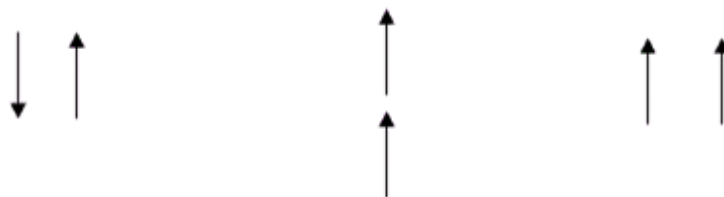


Figure: a) stray-field of a dipole, b) interaction between two dipoles

$$U(\mathcal{M}) = E_{\text{Exchange}}(\mathcal{M}) + E_{\text{Aniso}}(\mathcal{M}) + E_d(\mathcal{M})...$$

- Description: The stray field term describes the energy of the magnetostatic interaction of dipoles via the magnetostatic field.
- Energy of a second dipole in this field:



- Micromagnetic approximation: Transition from discrete spins to the continuous magnetization:
- In a solid:  $\nabla \mathbf{B} = \nabla(\mu_0(\mathbf{H}_d + \mathbf{M})) = 0$  (Maxwell Equation)
- Without external field:  $\nabla \mathbf{H}_d = -\nabla \mathbf{M}$

Energy of the stray field/self-energy of the magnetization in its field:

- Calculation of stray fields:
- Analogous to the calculation of the electrical field of a charge distribution.

Volume charges:  $\rho = \nabla \cdot \mathbf{M}$ , surface charges:  $\sigma = \mathbf{n} \cdot \mathbf{M}$

- Most often the integral for  $H_d$  is difficult to calculate analytically.

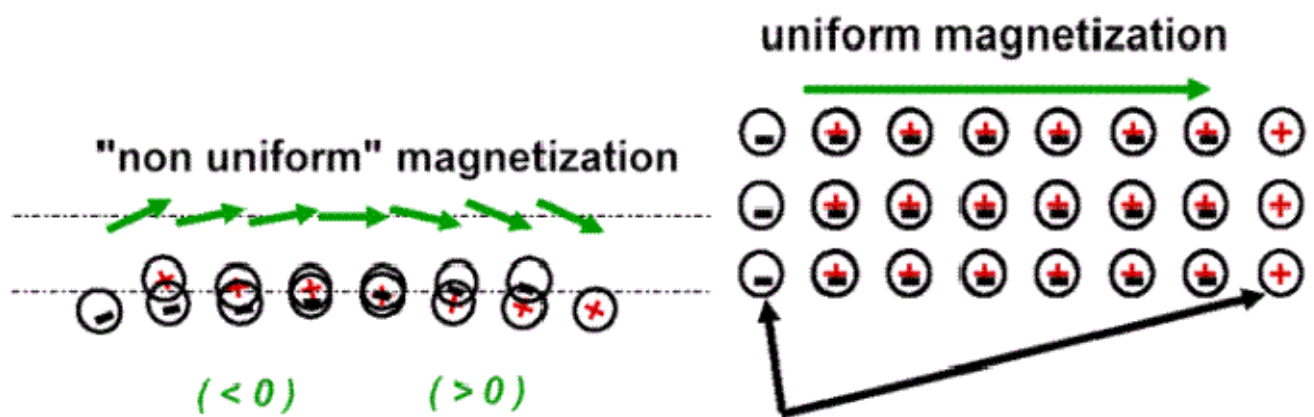
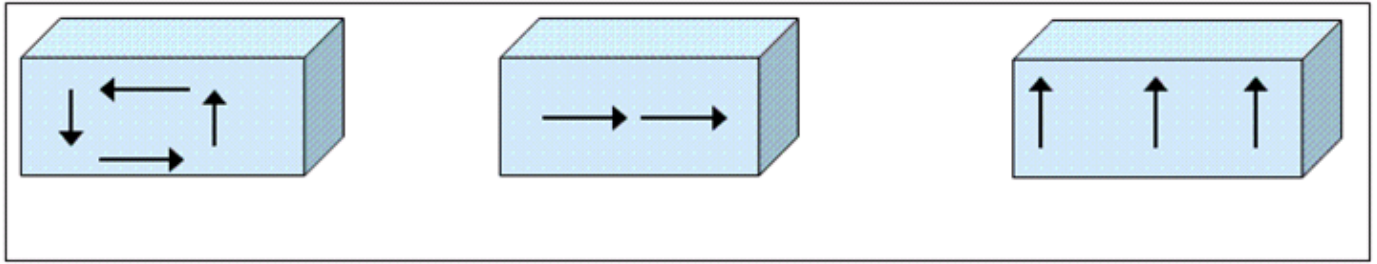


Figure: "Volume charges"  $\rho = \nabla \cdot \mathbf{M}$  (left), "surface charges"  $\sigma = \mathbf{n} \cdot \mathbf{M}$  (right)





$$\mathbf{H}_d = \int_{\Omega} \frac{(\mathbf{r} - \mathbf{r}')\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV - \int_{\delta\Omega} \frac{(\mathbf{r} - \mathbf{r}')\sigma(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dS$$

- Why is the left configuration ..... than the right ones even though it has volume charges but no surface charges?

- If one calculates:

And in particular the first integral over  $H_d^2$  over the whole space, one sees that for a configuration such as the one on the left,  $H_d$  exists only locally at the structure.

If one uses the far field approximation then the stray field is zero in the far field and the total integral is small.

- For the other two magnetization configurations one finds in the far field a dipolar field that drops off only slowly, which means the integral over the whole space is large!

### 3.5 Summary of the energy terms:

$$G = U(\mathcal{M}) = E_{\text{Zeeman}} \\ = E_{\text{Exchange}} + E_{\text{Aniso}} + E_d + E_{\text{Magnetostriktion}} + E_{\text{Stress}} + E_{\text{Zeeman}} \dots$$

- Summary:
- Thermodynamics: Gibbs free energy  $G$  is minimized in equilibrium.
- Micromagnetic approximation: transition from discrete spins to continuous magnetization  $\mathbf{M}(\mathbf{r})$ .
- **Partner-task: Draw for each energy term an energetically favourable and unfavourable magnetization configuration.**

### 3.6 Stoner-Wohlfarth model for single domain (monodomain) systems

- Simplest case of a system, in which all spins are oriented parallel.
- Realistic for which geometries?
- Scaling of energy terms: What happens with the energy-terms, if one scales down a given magnetization configuration?

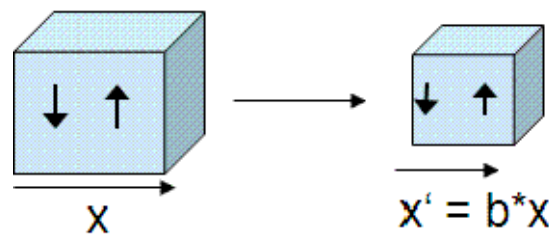
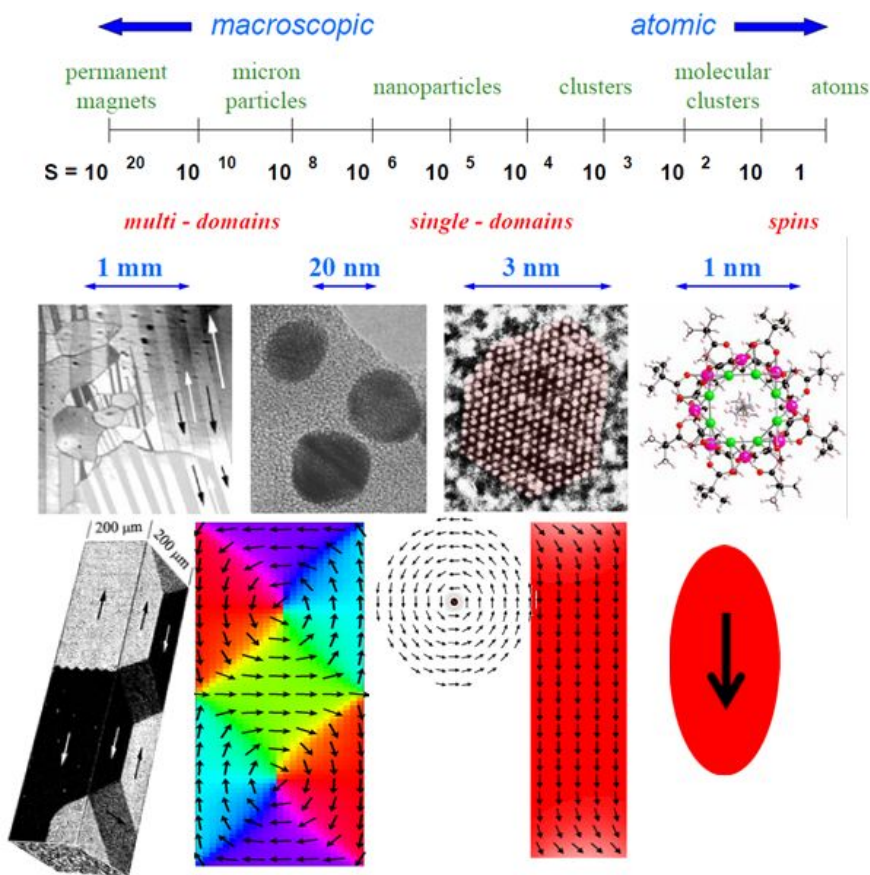


Figure: Reduction (scaling) of lateral sizes  $x \rightarrow x' = b*x$

- Apart from the exchange energy, all energy terms are scale invariant (energy proportions are size independent).

## 3.7 Overview of Magnetism on different Scales



### Large macroscopic systems:

Surfaces and edges do not play a role. Magnetization is homogeneously oriented (minimizing exchange energy). Defects often lead to chaotic inhomogeneous magnetization.

### Mesoscopic Systems

(>nm; <mm): Surfaces and edges govern the magnetization, which minimizes the stray field --> magnetization configuration is governed by the geometry.

### Microscopic systems:

Scaling of the exchange energy dominates. The magnetization is homogeneous and independent of the geometry. Ultimately small are single molecular magnets (details magnetism lecture course).

## 4 Micromagnetism in inhomogeneous systems

Imaging of the magnetization configuration shows inhomogeneous magnetization (in particular along the edges).

→ Stoner-Wohlfarth theory not applicable!

- With which model does one describe such inhomogeneous  $M$ ?
- Why does such an inhomogeneous magnetization configuration form?

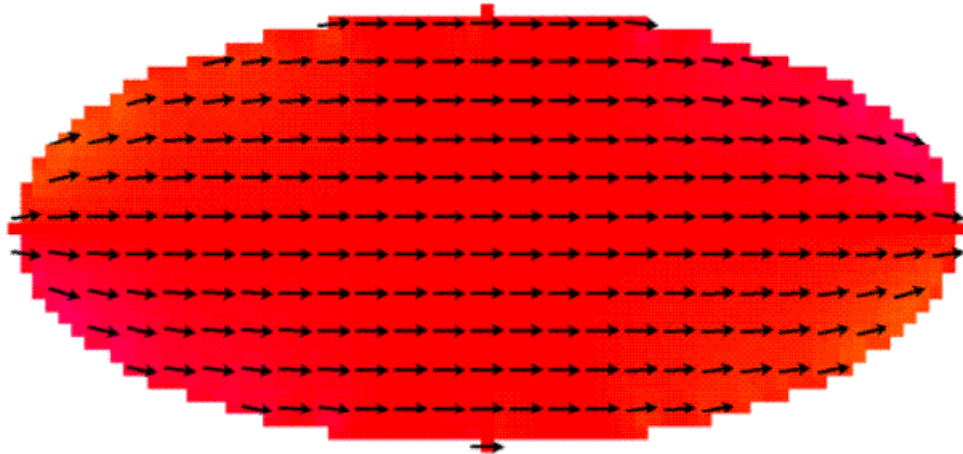


Figure: Magnetization configuration in an ellipsoid in a single domain state; the magnetization bends at the edges

## 4.1 Energy term and inhomogeneous M

- Which energy terms are reduced due to inhomogeneous magnetization?
- Zeeman energy? No!
- Exchange energy? No!
- Anisotropy energy? No!
- Stray field energy? Yes!

The stray field energy tries to align the magnetization parallel to the structure edges, and this leads to inhomogeneous magnetization.

## 4.2 Domain formation - calculation of energy G of a given configuration M

- Reduction of the stray field energy while at the same time the exchange energy is increased due to domain formation.

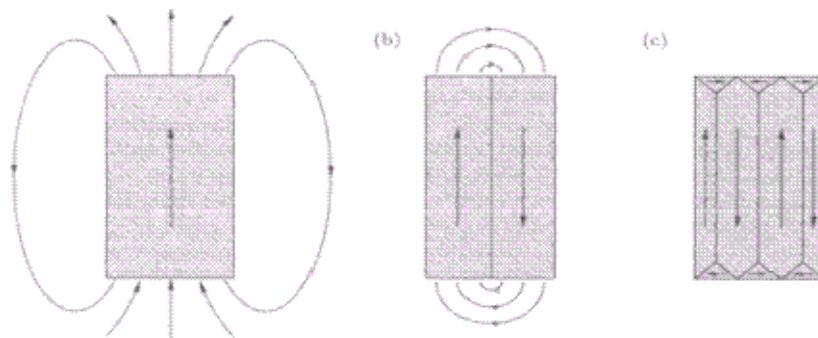


Figure: Reduction of stray field energy by formation of multiple domains with stray-field flux closure



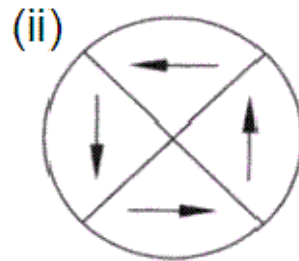
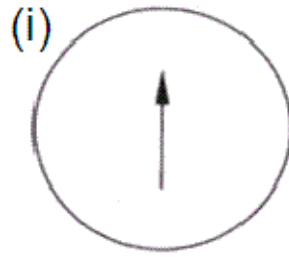


Figure: Monodomain and multidomain states in a sphere

- Ferromagnetic sphere with radius  $r$ .
- **(i)** The energy of a monodomain state is:  $G_i = E_d$ :

For a homogenously magnetized sphere (5.4.7):

$$N = -1/3:$$

$$(\text{sphere volume } V = 4/3\pi r^3)$$

- **(ii)** Reduction of the stray field energy ( $E_d$ ) while at the same time the exchange energy ( $E_A$ ) is increased due to domain formation.

The energy of the four quadrant state (ii) is:  $G_{ii} = E_{\text{Exchange}}$  (the stray field energy is nearly 0, but there is exchange energy at the domain boundaries (domain walls)):

(with  $\sigma_{\text{DW}}$  the energy per unit area of the domain wall in the sphere).

- The configuration (i) is more favorable than (ii), if:
- Due to the scaling of the energy terms, the monodomain state is energetically favourable for small structures. For realistic values one obtains  $r_{\text{crit}} = 10^{-7}\text{m}$ .

## 4.3 Domain formation - inhomogeneous M

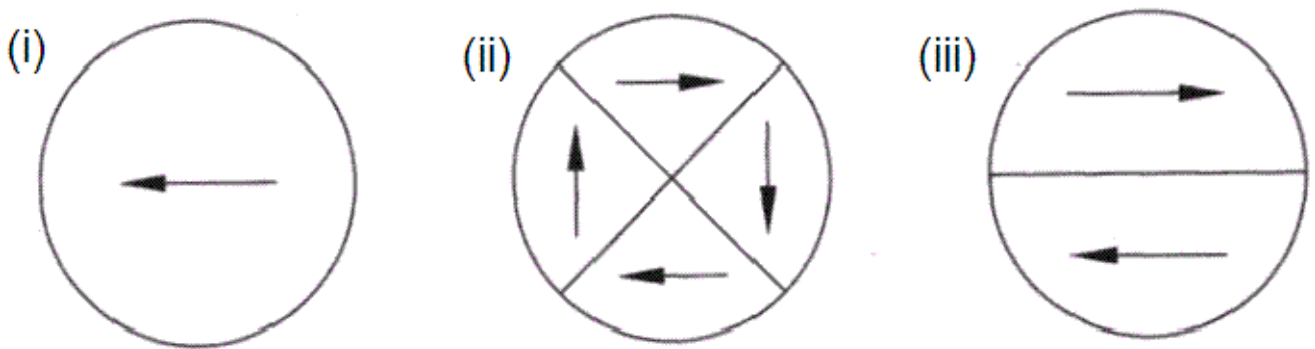


Figure: Possible magnetization configurations of a sphere

- We can now calculate for the different magnetization configurations (a)-(c), (i)-(iii) the energy and determine the energetically most favourable configuration.
- But have we guessed the configuration and possibly overlooked an even more favourable configuration?
- It would be better to directly calculate the optimum configuration, which can be done analytically for instance for a domain wall (see exercise)