Advance Organizer:

Chapter 1:

Introduction to **single** magnetic moments:

- Magnetic dipoles
- Spin and orbital angular momenta
- Spin-orbit coupling
- Magnetic susceptibility,

Magnetic dipoles in a magnetic field:

- Zeeman splitting

Chapter 2:

Introduction to ensembles of **interacting** magnetic moments:

- Dipolar coupling between moments
- Exchange coupling between moments - Ferromagnetism

Chapter 3:

Micromagnetism of nanoscale systems

- Static micromagnetism:
  Energy terms, homogeneous and inhomogeneous magnetization configurations
- Magnetodynamic switching due to magnetic fields

Literature:

- S. Blundell Magnetism in Condensed Matter
- J. Stöhr Magnetism – From Fundamentals to Nanoscale Dynamics
- B. Heinrich Ultrathin Magnetic Structures
- Aharoni Micromagnetism
- H. Kronmüller Micromagnetism and the Microstructure of Ferromagnetic Solids

1 Fundamentals

1.1 Units, definitions and fundamental equations

- How should we deal with **B** and **H** which are usually used for magnetic fields?

- We use the definition of the Lorentz[1] force $\mathbf{F}_L$ as the starting equation:

- This is valid for a particle with charge q and velocity **v** in **E** and **B** fields.

- In magnetism, one uses the magnetic induction **B** which is analogous to the electric field **E** in electrostatics. We agree that **B** from now on is called magnetic field. The unit of magnetic field is Tesla[2]:

- The unit of **H** is: $[\mathbf{H}] = 1\frac{A}{m}$

[1] Hendrik Antoon LORENTZ (1853-1928), Dutch physicist and mathematician

[2] Nikola TESLA (1856-1943), Russian inventor and engineer

- If a material is introduced into an **H** field, it often exhibits a magnetic moment. The magnetic moment per unit is called the magnetization **M**. The direction of the vector **M** gives the direction of the magnetic dipole moment. The relation between **H** and **M** is often nonlinear.
  A dimensionless proportionality factor, the susceptibility χ, appears:

- Important equations are the Maxwell[3] equations in matter:

(with **D**: dielectric displacement; **j**: current density; ρ: charge density)

[3] James Clerk MAXWELL (1831-1879), Scottish physicist

- The magnetization **M** now is related to the magnetic induction or the magnetic field **B** by:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

- Introducing the equation $\mathbf{M} = \chi\mathbf{H}$ into this we obtain:

(with $\mu_0 = 4\pi \cdot 10^{-7}\frac{N}{A^2}$: magnetic permeability of vacuum; μ: magnetic permeability of the material; μ_r: relative permeability)

- Sometimes one uses the magnetic polarization $\mathbf{J}_{pol}$:

$$\mathbf{J}_{pol} = \mu_0\mathbf{M}$$

- Which has the same dimensions as **B**.

- The potential energy of a magnetic dipole $\boldsymbol{\mu}_m$ in a magnetic field $\mathbf{B}_0$ is given by:

$$E_m = -\mu_m \cdot \mathbf{B}_0$$

- On this dipole a torque acts:

$$\tau = \mu_m \times \mathbf{B}_0$$

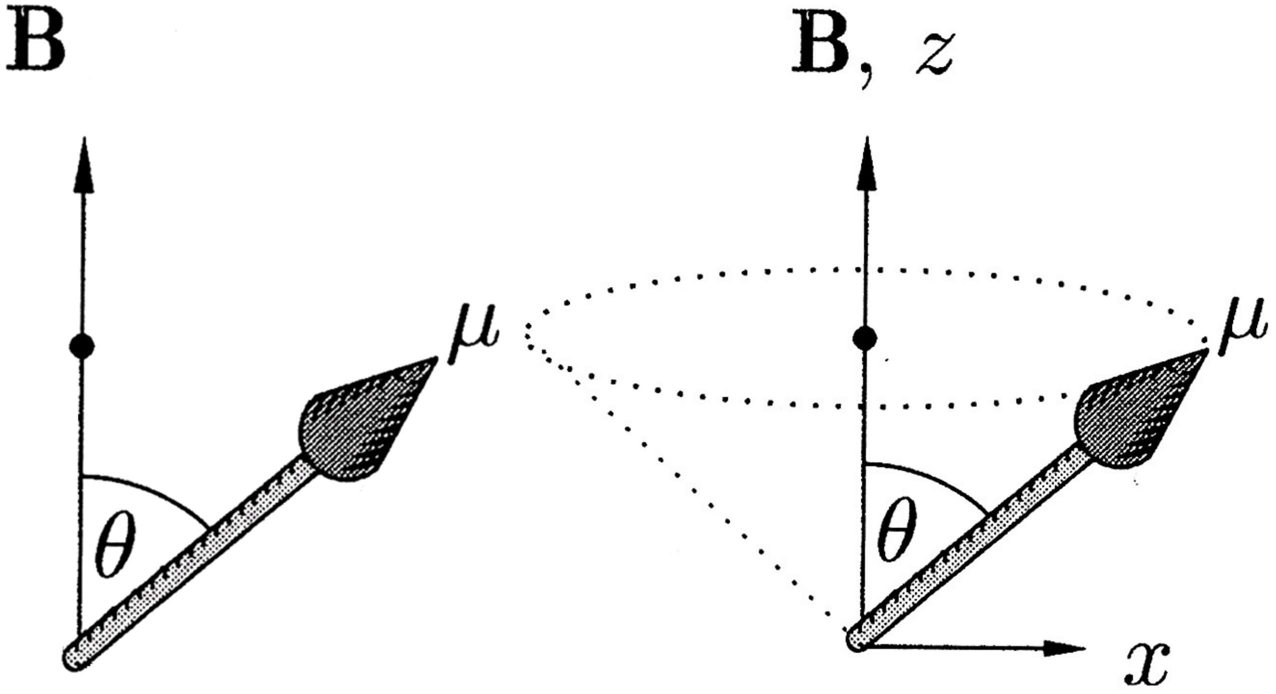

Figure: Potential energy of a magnetic dipole (left), and torque acting on dipole (right)

- In an inhomogeneous field, the magnetic dipole $\boldsymbol{\mu}_m$ feels the force:

- The magnetic field **B** generated by a magnetic dipole $\boldsymbol{\mu}_r$ at a distance **r** is:

$$\mathbf{B}(\mathbf{r}) = \mu_0\left(\frac{3(\mu_m \cdot \mathbf{r})\mathbf{r}}{r^5} - \frac{\mu_m}{r^3}\right)$$

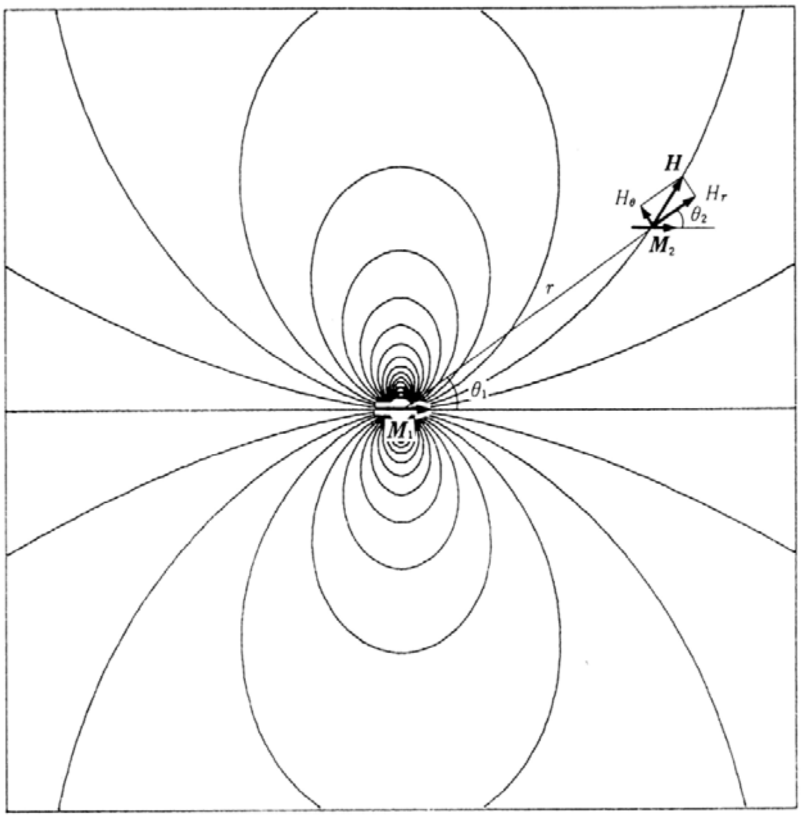

Figure: Fields of a magnetic dipole

- So for the interaction between two dipoles $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ at the positions $\mathbf{r}_1$ and $\mathbf{r}_2$ one obtains:

(with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$: the difference vector (distance))

1.2 The magnetic moment, magnetic susceptibility

- The atomic magnetic moment **μ** can be thought of as a circular current I of electrons around an area **A:**

$$\boldsymbol{\mu} = I \cdot \mathbf{A}$$

(with $\mathbf{A} = A \cdot \hat{\mathbf{n}}$, $\hat{\mathbf{n}}$: plane normal vector)

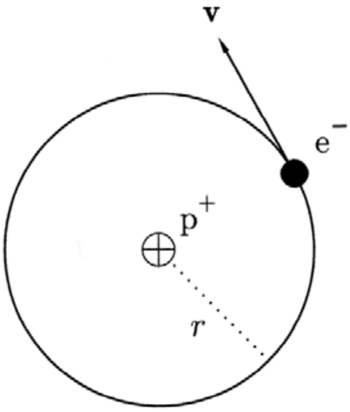

Figure: The electron of the hydrogen atom travels with velocity **v** in a circular orbit around the nucleus, which consists of a single proton

- From this follows:

$$\boldsymbol{\mu} = -\frac{1}{2}|e|\omega r^2 \hat{\mathbf{n}}$$

(with $A = \pi r^2$; ω: the angular frequency; $-\frac{e\omega}{2\pi}$: the circular current).

- If one compares this expression with the angular momentum **J** of an electron of mass m_e:

- This leads to:

- This is called the angular momentum character of the magnetic moment. The angular momentum, as well as the magnetic moment, are quantized units of ħ. The Bohr[4] magneton μ_B constitutes the smallest magnetic dipole moment of an electron, which cannot be broken up into smaller parts:

$$\mu_B = \frac{|e| \cdot \hbar}{2m_e} = 9,27 \cdot 10^{-24} \frac{J}{T}$$

[4] Niels BOHR (1885-1962), Danish physicist

1.2.1 Orbital and spin moment

- Electrons have an orbital moment and a spin moment.
  - Electrons carry out an orbital motion around the positively charged nucleus. From this one obtains an orbital moment:

$$\mu_l = -\frac{|e|}{2m_e}\mathbf{l}$$

(with $\mathbf{l}^2 = l(l+1)\hbar^2$)

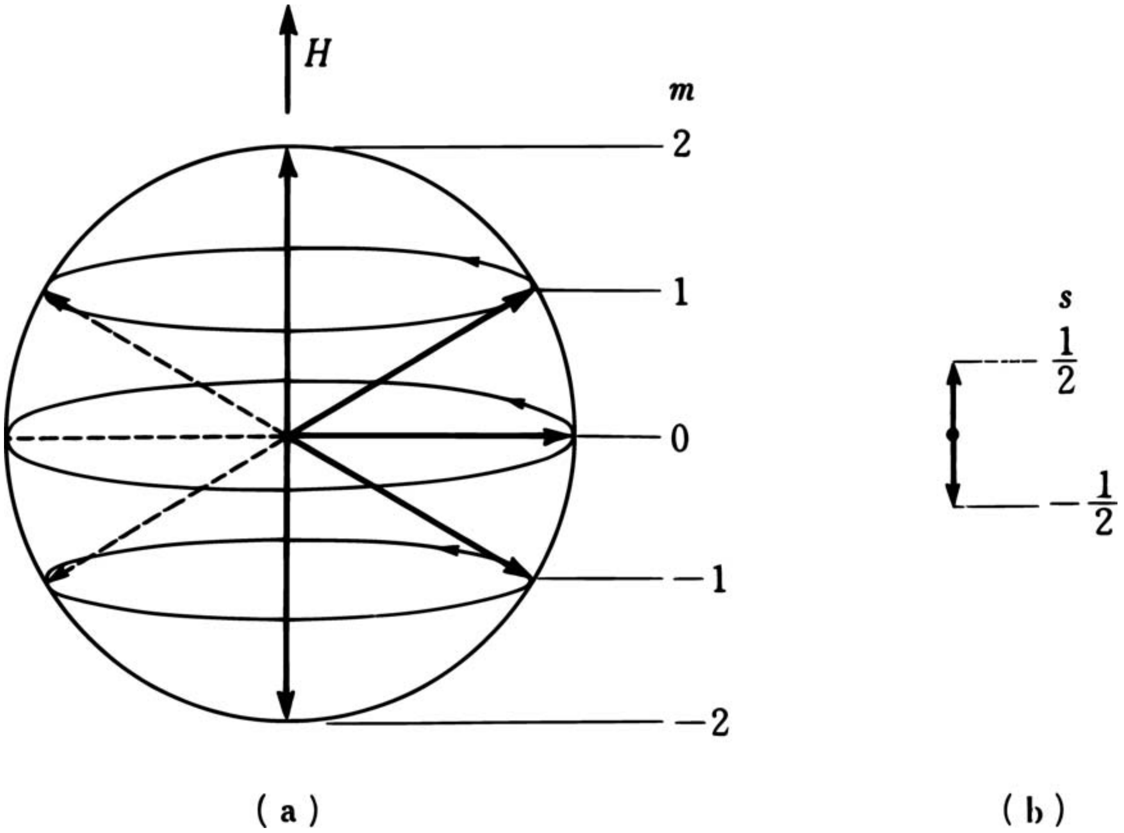

Figure: Projection of **l** onto the magnetic field direction.

- Electrons also have a spin moment (relativistic effect), which can be considered as a rotation of itself around an axis which is given by the magnetic field. This leads to the spin moment:

$$\mu_s = -g_e \frac{|e|}{2m_e} \mathbf{s}$$

$$\text{(with} \quad s^2 = s(s+1)\hbar^2\text{)}$$

- Both these angular motions are related to magnetic moments.

- The spin moment is parallel or antiparallel to the direction of an external magnetic field (z - direction) and has the amplitude:

$$\mu_s = g_e \frac{|e|}{2m_e} |s_z| \hbar = g_e \mu_B |s_z| \approx \mu_B$$

- s_z can take the values of ±½.

Note: In an external magnetic field, a parallel alignment is energetically most favourable. This is the origin of paramagnetism due to spin moments. The orbital moment is weakened by the application of an external magnetic field according to the Lenz[5] rule, since the external field induces a counter current (diamagnetism).

- **l** and **s** are themselves not good quantum numbers, but **j** is.

- The component of the magnetic moments **μ** parallel to **j** is conserved:

(with g_j: Landé factor for the eigenvalue $j\hbar$ of the angular momentum **j**)

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

- The amplitude of the total magnetic moment is given by:

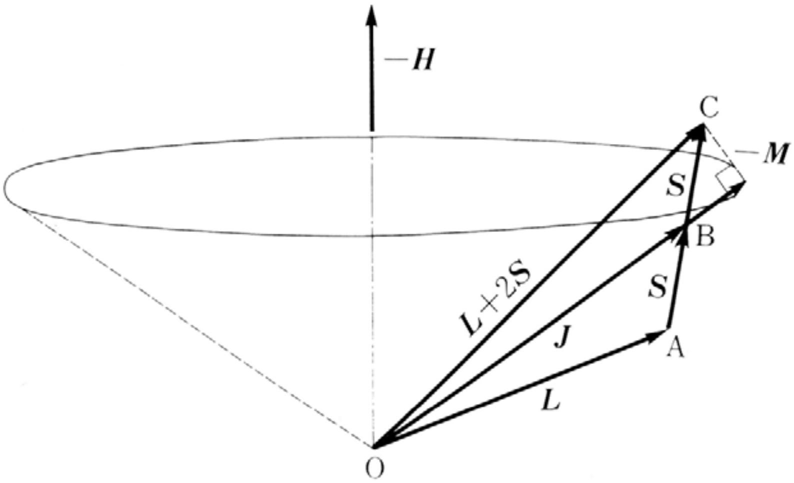

Figure: **LS** coupling

[5] Heinrich LENZ (1804-1865), Baltic physicist

1.2.2 The Einstein-de Haas effect

- The observation of the Einstein[6]-de Haas[7] effect is the proof that the magnetization is related to an angular momentum, showing that:

$$\mu_j = -g_j \frac{|e|}{2m_e}\mathbf{j}$$

- Design of the experiment:
  - A magnetic sample is suspended from a quartz wire, which is attached to a rotatable mirror.
  - The rotation of the sample is detected by the laser reflection in the mirror.
  - Magnetizing the sample is achieved by an external magnetic field, which means a change of the angular momentum.
  - Due to the conservation of angular momentum the crystal lattice has rotated in the opposite direction to compensate for the change in the angular momentum due to the change in the magnetization.
  - By resonant excitation due to an external AC field, the angular momentum can be measured.
  - For a known magnetization one can therefore determine g.

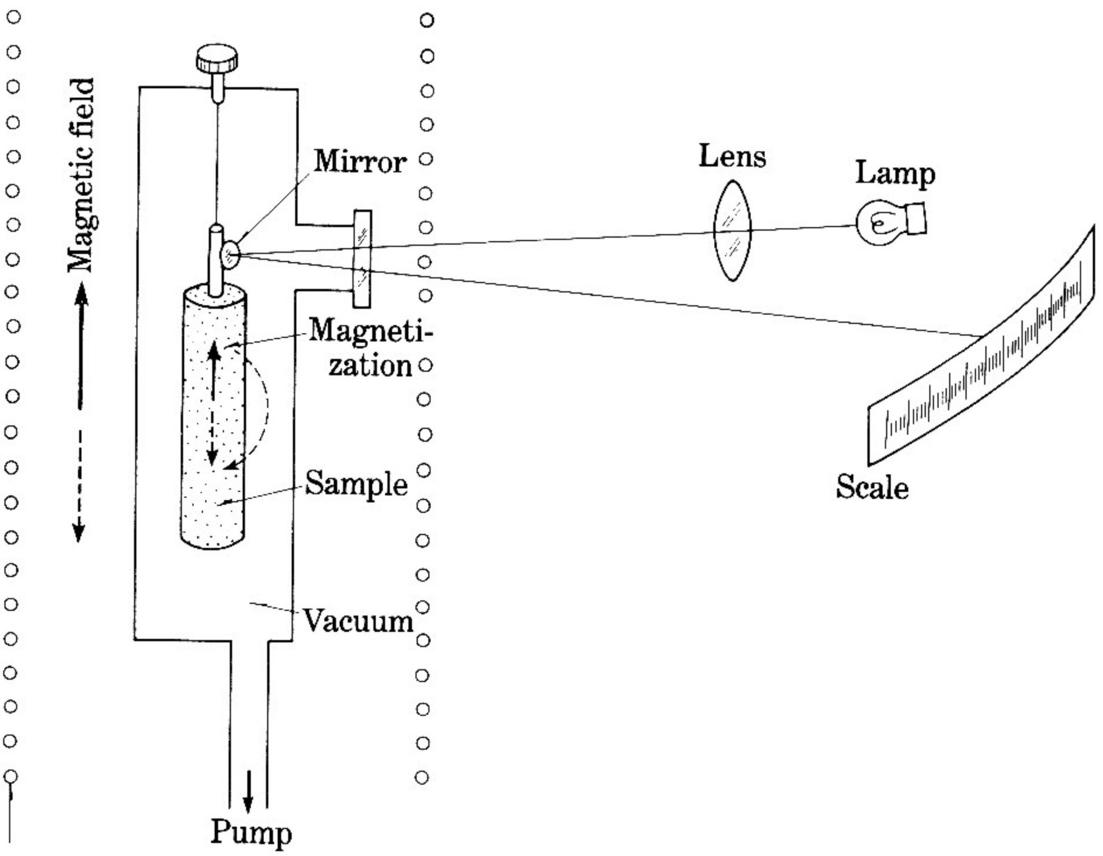

Figure: Experimental set-up for measurement of the Einstein-de Haas effect

[6] Albert EINSTEIN (1879-1955), German physicist

[7] Wander DE HAAS (1878-1960), Dutch physicist and mathematician

1.2.3 Susceptibility measurements (magnetization measurements)

- To measure the magnetic susceptibility one can use, for instance, a Faraday[8] balance.

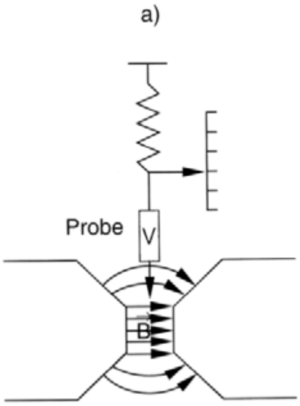

Figure: Schematic of a Faraday balance

- A sample with magnetization M feels in an inhomogeneous magnetic field a force:

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[8] Michael FARADAY (1791-1867), English physicist and chemist

- Verification by dimensional analysis:

$$\frac{d}{dz}\left(m^3 \cdot \frac{N}{A^2} \cdot \frac{A^2}{m^2}\right) = \frac{d}{dz}(N \cdot m)$$

- Faraday balances can be very sensitive, for instance $\Delta\chi \sim 10^{-10}$. Paramagnetic samples (with χ > 0) are pulled into the magnetic field, and diamagnetic samples (χ < 0) are expelled.

1.3 Spin-orbit coupling

- Spin-orbit coupling couples the spin and orbital momentum to a total angular momentum **j** = **l** + **s**. This coupling is described by the following energy term:

$$E_{ls} = \xi(\mathbf{r}) \cdot \mathbf{l} \cdot \mathbf{s}$$

(with $\xi(\mathbf{r}) = \frac{|e|}{2m_e c^2} \frac{1}{r} \frac{dV}{dr}$, V: Coulomb potential of the atomic nucleus)

Interpretation: in the rest system of the electron, the positively charged atomic nucleus moves and generates a magnetic field with which the spins of the electrons interact.

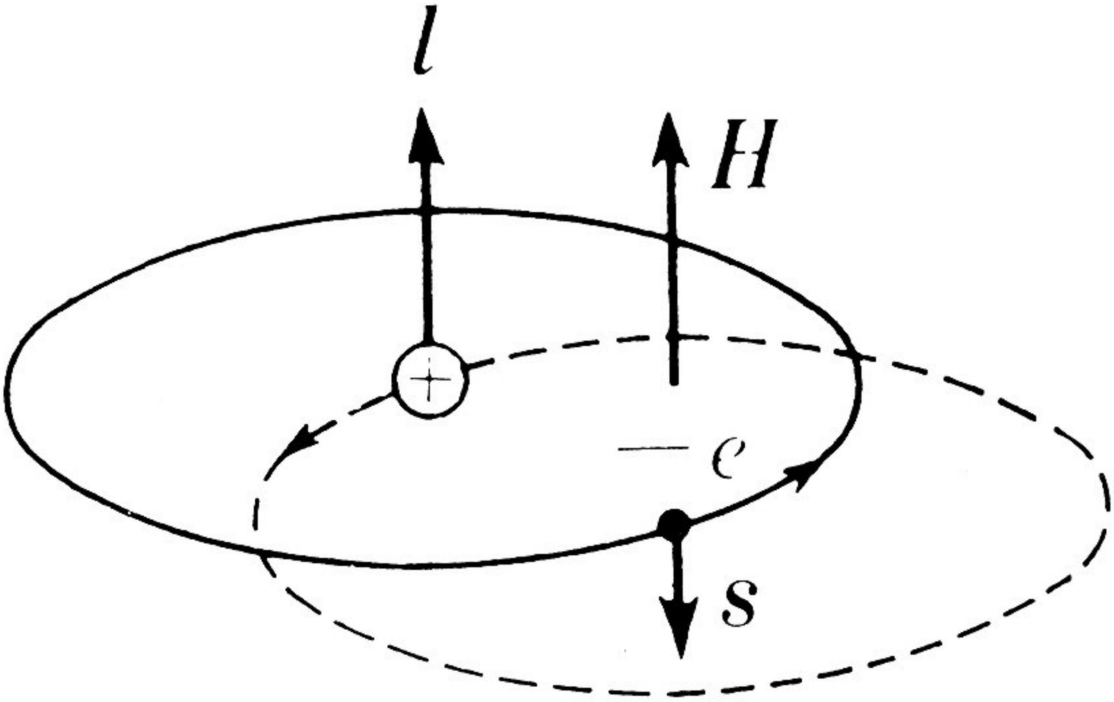

Figure: Spin-orbit interaction

- The transformation equations are:

$$\mathbf{E}' = \mathbf{E} + (\mathbf{v} \times \mathbf{B})$$
$$\mathbf{B}' = \mathbf{B} + \frac{1}{c^2}(\mathbf{E} \times \mathbf{v})$$

- **E** and **B** are the fields in the rest frame of reference of the atomic nucleus; $\mathbf{E} = \frac{Ze}{4\pi\epsilon_0 r^3}\mathbf{r}$. Inserting this yields:

$$E_{ls} = -\mathbf{B}' \cdot \mu_s = \frac{Ze^2}{4\pi\epsilon_0 c^2 m_e^2 r^3}(\mathbf{l} \cdot \mathbf{s})$$

- The electrostatic Coulomb[9] potential exhibits close to the nucleus a very high gradient. For large atomic numbers, dV/dr is particularly important.

[9] Charles DE COULOMB (1736-1806), French physicist

a) LS coupling (Russell-Saunders coupling)
Here dV/dr is small, so the orbital and spin moments couple amongst themselves much more strongly.

$$\mathbf{L} = \sum_{i=1}^{Z} \mathbf{l}_i; \qquad \mathbf{S} = \sum_{i=1}^{Z} \mathbf{s}_i; \qquad \mathbf{J} = \mathbf{L} + \mathbf{S}$$

(with Z: atomic number = electron number)

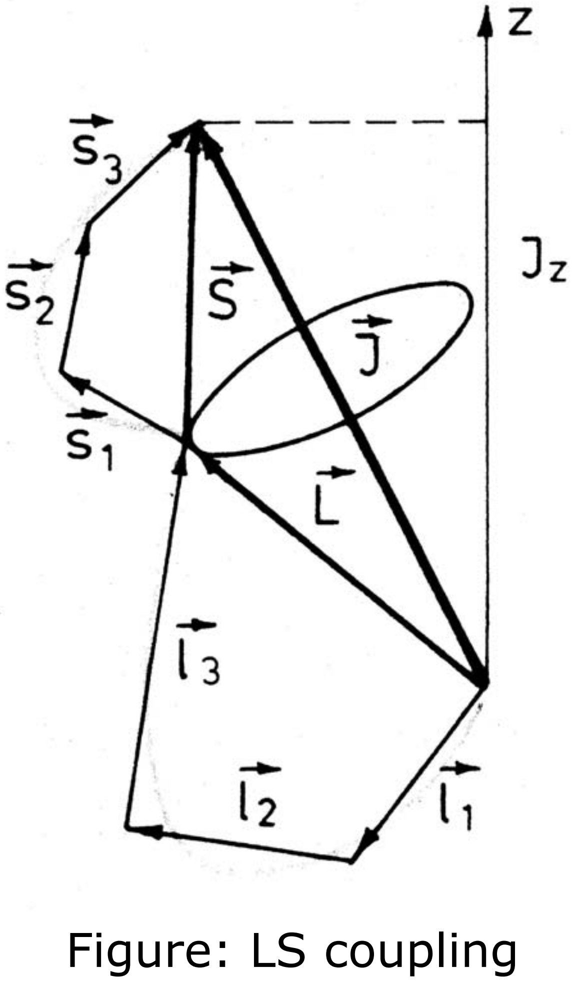

Figure: LS coupling

b) j-j coupling

For j-j coupling dV/dr is large, so first the orbital and spin moments of each electron couple:

$$\mathbf{j}_i = \mathbf{l}_i + \mathbf{s}_i; \qquad \mathbf{J} = \sum_{i=1}^{Z} \mathbf{j}_i$$

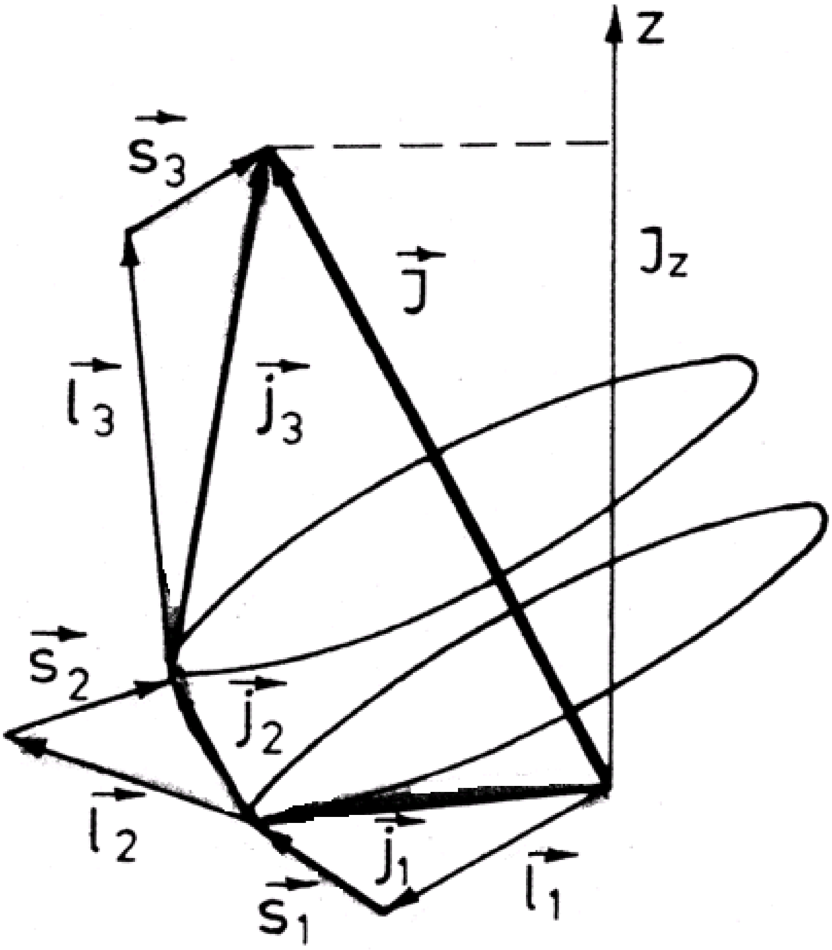

Figure: j-j coupling

Note: the spin-orbit interaction is at the origin of the magnetocrystalline anisotropy, discussed later, which is the preferred direction for the magnetization along certain crystallographic directions. Due to the spin-orbit interaction, spins couple to the crystal lattice. A rotation of the spins means, due to the spin-orbit interaction, also a rotation of the orbital moments. There is a change in the overlap between wave functions of neighbouring atoms.

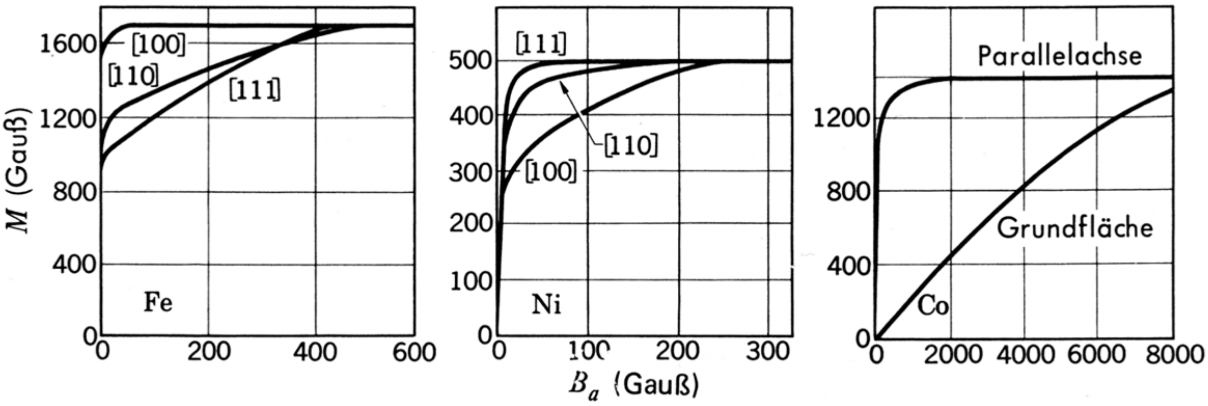

Figure: Magnetization curves for single crystals of Fe, Ni and Co

1.4 The Hund rules

- The Hund[10] rules are rules for filling of the electronic states in an atom step by step. The Pauli[11] principle prohibits the multiple occupation of an electronic state (every electronic state can be occupied with two electrons with spin quantum number ± ½). The aim is to determine the most energetically favourable configuration.
- Relevant for this are:

  - **Coulomb energy**

    Total spin S is maximum: this minimizes the possibility of having two electrons at the same position, which reduces the Coulomb energy.

  - **Antisymmetric total wave function**

    The orbital moment l is maximum (as long as compatible with S): the larger the orbital moment, the further the electrons are, on average, from the nucleus, and therefore the larger the average distance between them. This also reduces the Coulomb energy.

[10] Friedrich HUND (1896-1997), German physicist
[11] Wolfgang PAULI (1900-1958), Austrian-American physicist

- **Spin-orbit coupling energy**

  The total angular momentum J is minimum for electron shells which are filled less than half, and maximum for electron shells which are filled more than half. This means that the spin orbit coupling energy has the smallest possible value.

1.5 Zeeman splitting

- In an external magnetic field **B**$_0$ the direction of quantization corresponds to the direction of the effective field. Without an external field the total angular momentum J is (2J + 1) times degenerate. These states are numbered with the magnetic quantum number m_J, where m_J is in the range of –J to J:

$$-J \leq m_J \leq J$$

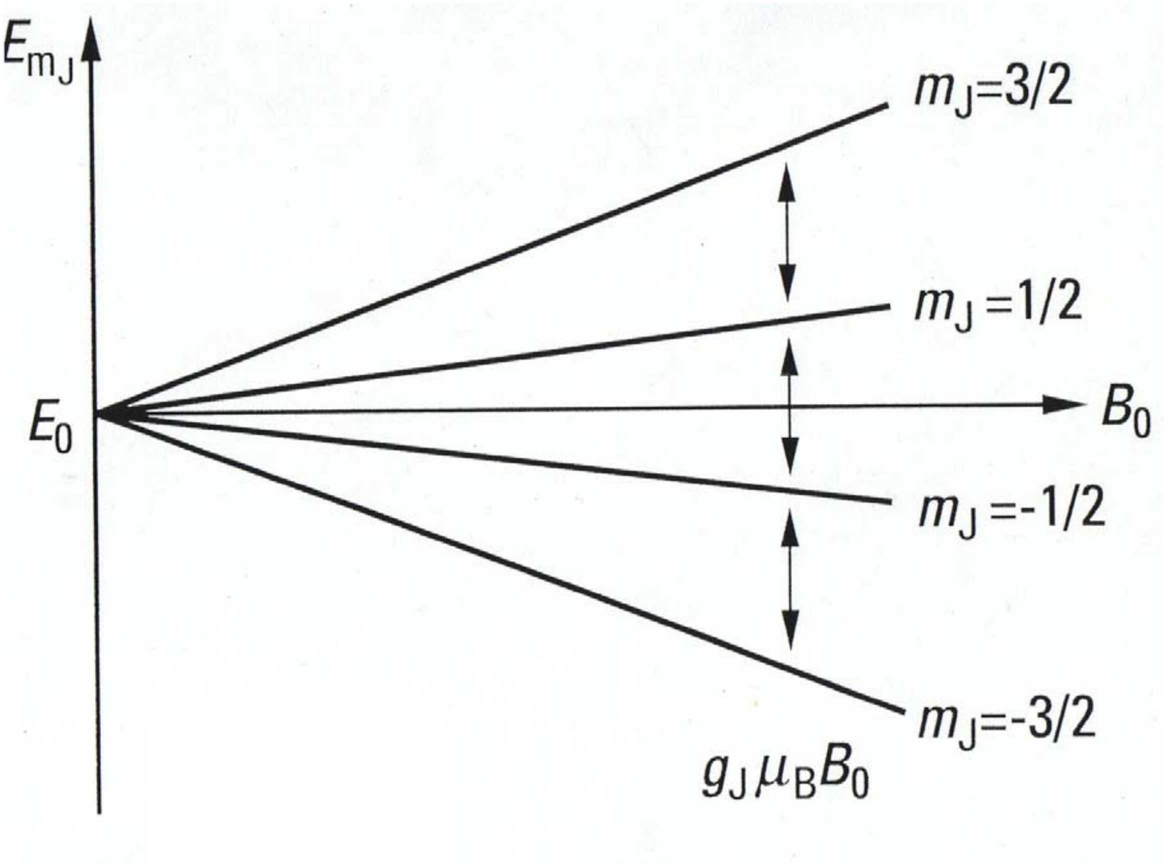

Figure: Zeeman splitting

- Along the quantization axis, the magnetic moment μ_{m_J} that corresponds to the state m_J is given by:

- In an external magnetic field B_0 which is weak compared to the strength of the spin orbit coupling, the energy degeneracy is lifted. There are 2J + 1 energy eigenvalues, and for the energy we find:

- Here the energy E_0 is the energy eigenvalue without an external field.